Appendix B

Derivation of AD2S93 Component Values

This Appendix discusses the methodology behind the selection of the component values for the AD2S93 and borrows heavily from work performed by Mark Schirmer at Analog Devices Assembled Products Division.

Based on the simplified system model in Figure B-1, expressions for the discrete components of the Loop Compensator of the AD2S93 can be derived.

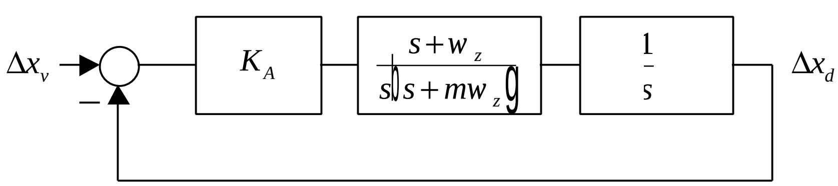

Figure B-1: Simplified Block Diagram of AD2S93

Collecting the gains of the different blocks of the AD2S93, the acceleration constant, K_A , is defined in Equation (B.1).

$$K_A = K_{ACRB} K_{BPF} K_{PSD} K_{INT} K_{VCO} \tag{B.1}$$

where the constituent gains are defined in Table B-1:

Term	Definition
K_{ACRB}	AC Ratio Bridge Gain
K_{BPF}	Band Pass Filter Gain
K_{PSD}	Phase Sensitive Demodulator Gain
K_{INT}	Integrator Gain
K_{VCO}	Voltage-Controlled Oscillator Gain

Table B-1: System Component Definitions

The open loop transfer function is expressed:

$$G_{E_{OL}}(s) = K_A \frac{s + w_z}{s^2 (s + m w_z)} \tag{B.2}$$

where

$$w_z = \frac{1}{R_2 C_2} \tag{B.3}$$

$$m = \frac{C_1 + C_2}{C_1} \tag{B.4}$$

where the R_2, C_1 and C_2 are the discrete components that make up the Analog Devices compensator. To maximize stability, the gain of the open loop system is set to 1.0 at the point of maximum phase lead. Equivilently, when $s = j\sqrt{mw}_z$, the magnitude of Equation (B.1) is defined

$$\left| G_{E_{OL}}\left(j\sqrt{mw}_z \right) \right| = 1 = \frac{K_A}{m\sqrt{mw}_z^{\,2}} \tag{B.5}$$

Therefore, the closed loop system can be rewritten in the form of Equation (B.6).

$$\begin{aligned} G_{E_{CL}}(s) &= K_A \frac{s + w_z}{s^2 (s + m w_z) + K_A (s + w_z)} \\ &= K_A \frac{s + w_z}{s^3 + m w_z s^2 + K_A s + K_A w_z} \end{aligned} \tag{B.6}$$

If the following substitutions are made in Equation (B.6),

$$K_A = m\sqrt{mw}_z^{\,2} \tag{B.7}$$

$$x = \frac{\sqrt{m} - 1}{2} \tag{B.8}$$

$$s_N = \frac{s}{w_z} \tag{B.9}$$

Therefore, the transfer function in Equation (B.6) can be expanded:

$$\begin{aligned} G_{E_{CL}}(s) &= \frac{m\sqrt{m}(s_N + 1)}{s_N{}^3 + ms_N{}^2 + m\sqrt{m}s_N + m\sqrt{m}} \\ &= \frac{m\sqrt{m}(s_N + 1)}{(s_N + m)\left[s_N{}^2 + 2x\sqrt{m}s_N + m\right]} \\ &= \frac{\sqrt{m}(s_N + 1)}{(s_N + m)} \frac{m}{s_N{}^2 + 2x\sqrt{m}s_N + m} \end{aligned} \tag{B.10}$$

The transfer function $G_{E_{CL}}(s)$ is a Type II system in series with a lead compensator. The resonant frequency of this system is $\sqrt{m}$. Optimal tracking is therefore assumed to occur when the damping factor $x = 0.707$. Using Equation (B.8), the value of m can be determined.

$$m = 5.83 \tag{B.11}$$

Using the assumption that bandwidth is the 3dB point on the normalized frequency graph, the relationship for w_z with respect to the desired bandwidth.

$$w_{-3\text{dB}} = 4w_z \tag{B.12}$$

From Equations (B.11) and (B.12), the component values for the system may be determined.

Equations (B.4), (B.8), and (B.11) are used to derive a relationship between C_1 and C_2:

$$C_2 = 4.83C_1 \tag{B.13}$$

From Equations (B.1) and (B.7), the following relationship is obtained between the desired bandwidth and C_1.

$$\begin{aligned} K_A &= m\sqrt{m}w_z{}^2 = K_{ACRB}K_{BPF}K_{PSD}K_{INT}K_{VCO} \\ (5.83\sqrt{5.83})&\left[\left(\frac{2pf_{-3\text{dB}}}{4}\right)\right]^2 = (4.1x10^{-3})\left(\frac{2\sqrt{2}}{p}\right)\frac{1}{3}\frac{1}{(R_1\|R_7)C_1}K_{VCO} \\ \Rightarrow C_1 &= \frac{3.542x10^{-5}K_{VCO}}{(R_1\|R_7)f_{-3\text{dB}}} \end{aligned} \tag{B.14}$$

where $R_1 \| R_7$ is the input impedance to the compensator. The next step is to apply the following component constraints based on the internal structure of the AD2S93:

- $R_1 = 25k\Omega$ (internal resistor),
- R_7 is recommended to be set at $15k\Omega$, and
- K_{VCO} is either $640K\Omega$ or $160K\Omega$, depending on desired frequency range

Applying these constraints, the expression for C_1 can be rewritten for Mode 1 and Mode 2:

$$C_1 = \frac{1}{414 f_{-3\mathrm{dB}}{}^2} \quad \text{for Mode 1} \tag{B.15}$$

$$C_1 = \frac{1}{1655 f_{-3\mathrm{dB}}{}^2} \quad \text{for Mode 2} \tag{B.16}$$

where Mode 1 is for bandwidths from 500 Hz to 1.25 kHz ($K_{VCO} = 640K\Omega$), and Mode 2 is for bandwidths from 145 Hz to 500 Hz ($K_{VCO} = 160K\Omega$).

Given Equations (B.3), (B.13), (B.15) and (B.16), the values for R_2 are obtained.

$$R_2 = \frac{4}{2pf_{-3\mathrm{dB}}} \frac{414 f_{-3\mathrm{dB}}{}^2}{4.83} = 55 f_{-3\mathrm{dB}} \quad \text{for Mode 1} \tag{B.17}$$

$$R_2 = \frac{4}{2pf_{-3\mathrm{dB}}} \frac{1655 f_{-3\mathrm{dB}}{}^2}{4.83} = 218 f_{-3\mathrm{dB}} \quad \text{for Mode 2} \tag{B.18}$$