Chapter 3
Torque Sensor

This chapter characterizes the issues surrounding the development of the torque sensor, specifically addressing measurement methods, transducer technology and converter technology. In addition, the system equations of the sensor will be formulated for use in control design.

3.1 Measurement Methods

In torque sensor designs, researchers have used a variety of methods to infer the torque applied to a link. Specifically, two basic approaches are employed.

3.1.1 Open Loop or Indirect Methods

Open loop methods often infer the estimated value of the torque applied to the link by relying on *a priori* knowledge of joint parameters such as the link, motor, and transmission terms and sensing the applied voltages and currents. Since most models suffer from parameter error or simplified assumptions, the joint torque estimate is often erroneous. For example, the relationship of current to the applied torque is often determined by the following linear equation:

$$t_{applied} = K_T I \tag{3.1}$$

where K_T is the torque constant of the motor and I is the current applied to the motor. Many applications use Equation (3.1) and a measurement of current to approximate the torque applied to a link. This method is imprecise since it does not take into account the complexities of the gearing mechanism, non-linear effects of friction, back EMF and backlash - all of which contribute to loss of torque from the motor to the link. In models that do include these effects, the complexity and computational power required often reduces sensor performance to bandwidths on the order of 10 to 100 Hz.

3.1.2 Feedback or Direct Methods

Feedback methods require some form of transducer attached to the motor shaft to sense the torque applied. These methods typically involve sensing some form of displacement or strain applied by the motor to some structure, such as a shaft or a flexible member. In these methods, the connecting member is designed in way that when torque is applied, an angular displacement or strain results on the member. Transducers such as strain gages, piezoceramics, or optical fiber are used to measure strains on a member attached to the shaft, whereas inductive transducers, Hall Effect sensors or encoders are used for displacement measurements. Since these methods are based on more direct readings of the torque applied to the member, very little computation power required to determine the torque. Therefore, the measurement of the applied torque is often at a higher bandwidth and with higher performance characteristics and greater accuracy than open loop methods.

3.2 Mechanical Sensor

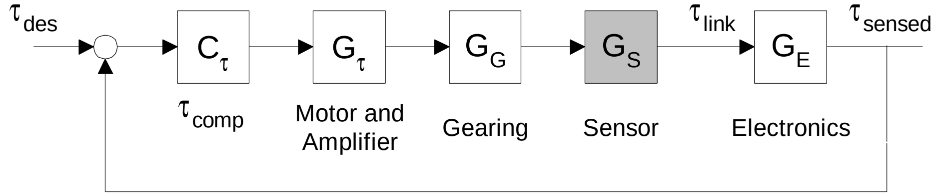

Figure 3-1: Generalized Joint Torque Loop

Based on previous design efforts [Vischer90], the ARTISAN wrist is equipped with a torsion spring between the transmission system and the link itself. The six-beam structure shown in Figure 3-2 allows flexibility about the z-axis, normal to the face of the sensor while providing greater stiffness along the x and y-axes. This flexibility in the axis of interest allows for a small, measurable displacement that can be tracked in order to measure the torque applied to the link. High stiffness along the other in-plane directions reduces the disturbance effects caused by radial forces, which improves sensor accuracy. Through this displacement, the torque being applied to the link can be sensed directly.

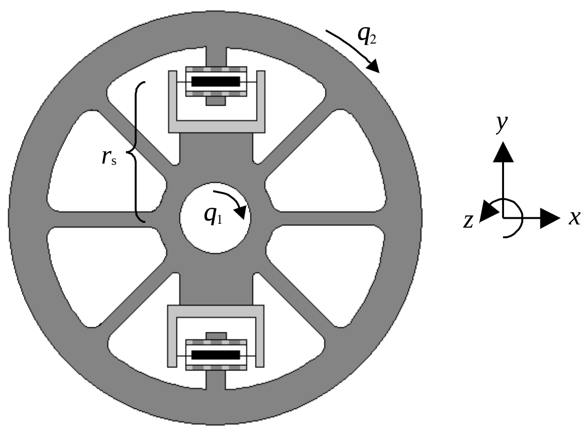

Figure 3-2: Six Spoke Sensor

The relationship between the sensed torque, t_s, and the angular deflection q_s is defined as:

$$t_s = k_s q_s \tag{3.2}$$

where

$$q_s = q_2 - q_1 \tag{3.3}$$

and k_s is the torsional stiffness of the spoked structure. The angular deflection q_s of the spokes is used to calculate t_s by using the small angle approximation of $dq_s \approx dx_s$, where x_s is the linear displacement of the LVDT core along the x-axis, at a distance r_s from the center of the sensor as shown in Figure 3-2. For small deflections dq_s, the equation becomes

$$t_s = \frac{k_s}{r_s} dk_s \quad (3.4)$$

The sensor’s torsional stiffness, k_s, is determined by Equation (3.5)

$$k_s = 4NEI\left(\frac{1}{l} + \frac{3r}{l^2} + \frac{3r^2}{l^3}\right) \quad (3.5)$$

where r is the inner radius of the sensor, N is the number of spokes, I is the modulus of the spoke sections, l is the spoke length, and E is Young's modulus.

3.2.1 Mechanical Sensor System Equation

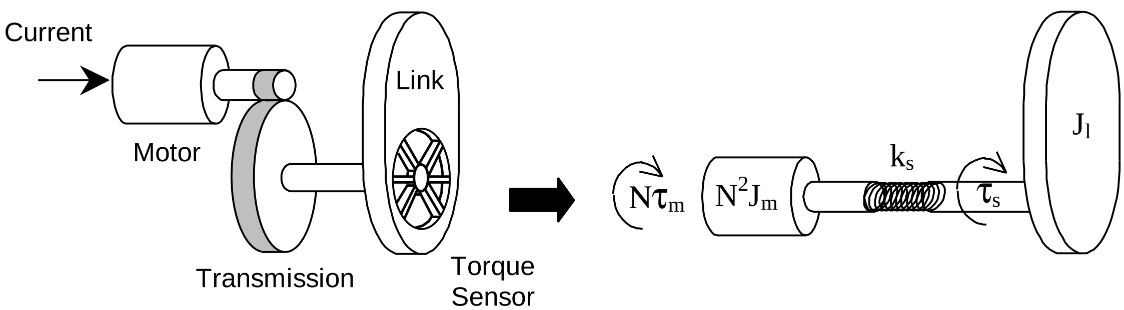

Figure 3-3: System Diagram of Mechanical Sensor

Since the torque sensor is a designed flexibility in the transmission of torque from the motor to the link, it can be represented as a linear spring as shown in Figure 3-3.

For the transmission system, the motor deflection q_m is defined as:

$$q_m = -Nq_s - Nq_l \quad (3.6)$$

where N is the gear ratio, q_s is the angular deflection of the sensor and q_l is the angular deflection of the link. By summing the forces on the link, the torque applied to the link, which is equivalent to the torque applied to the sensor, t_s, is defined as:

$$t_s = J_l \ddot{q}_l \quad (3.7)$$

where J_l is the rotational mass moment of inertia and $\ddot{q}_l$ is the angular acceleration of the link. Assuming zero initial conditions, the following transfer function results:

$$\frac{q_l}{t_s} = \frac{1}{J_l s^2} \quad (3.8)$$

where all of the terms have been defined previously.

Summing the forces on the rotor provides an expression for the torque applied by the motor, t_m, in terms of angular deflections.

$$N t_m = N J_m \ddot{q}_m + N d_m \dot{q}_m - d_s \dot{q}_s - t_s \quad (3.9)$$

where J_m is the rotational mass moment of inertia of the motor, d_m is the damping coefficient of the motor and d_s is the damping coefficient of the sensor. Assuming zero initial conditions, Equation (3.9) leads to the following result:

$$N t_m = N J_m q_m s^2 + N d_m q_m s - d_s q_s s - t_s \quad (3.10)$$

From Equations (3.2), (3.6), (3.8) and (3.10), the transfer function between the applied motor torque t_m and the sensed torque t_s can be derived:

$$\frac{t_s}{t_m} = \frac{b_0 s}{a_0 s^3 + a_1 s^2 + a_2 s + a_3} \quad (3.11)$$

where

$$b_0 = k_s \quad (3.12)$$

$$a_0 = J_m N \quad (3.13)$$

$$a_1 = d_m N + \frac{d_l}{N} \quad (3.14)$$

$$a_2 = \frac{J_m k_s N}{J_l} + \frac{k_s}{N} \quad (3.15)$$

$$a_3 = \frac{d_m k_s N}{J_l} \quad (3.16)$$

3.2.2 Free versus Fixed Configuration

For any configuration of the link, Equation (3.11) provides an effective expression for the transfer function. In the free configuration, the link inertia is determined by the inertia and configuration of the components found after the joint. In the fixed configuration, where the link is locked in a fixed position, the link inertia tends to infinity ($J_l \to \infty$). Therefore, the transfer function of the sensor in the fixed configuration becomes:

$$\frac{t_s}{t_m} = \frac{b_0}{a_0 s^2 + a_1 s + c_2} \tag{3.17}$$

where

$$c_2 = \frac{k_s}{N} \tag{3.18}$$

and the other terms are defined in Equations (3.12) through (3.14).

Through simple manipulations, Equation (3.17) can be rewritten in the standard second-order transfer function form.

$$\frac{t_s}{t_m} = \frac{N w_s^2}{s^2 + 2 z_s w_s s + w_s^2} \tag{3.19}$$

where

$$w_s = \sqrt{\frac{k_s}{J_m N^2}} \tag{3.20}$$

$$z_s = \frac{1}{2}\sqrt{\frac{d_m^2 N^2}{J_m k_s}} + \frac{1}{2}\sqrt{\frac{d_l^2}{J_m k_s N^2}} \tag{3.21}$$

Given the values for the terms listed in Equation (3.20) and Table 3-1, the damped resonant frequency of the mechanical sensor in the fixed configuration is approximately 130 Hz. This calculated frequency is a minimum bound on the resonant mode since the free configuration does not approximate the link inertia at infinity. This characteristic results in an additive component to the resonant frequency and an additional pole/zero pair close to zero on the s-plane.

Term	Value
k_s	9751 N-m/rad
J_m	80x10^{-6} kg-m^2
N	13.5

Table 3-1: Mechanical Sensor Values

3.3 Sensor Electronics

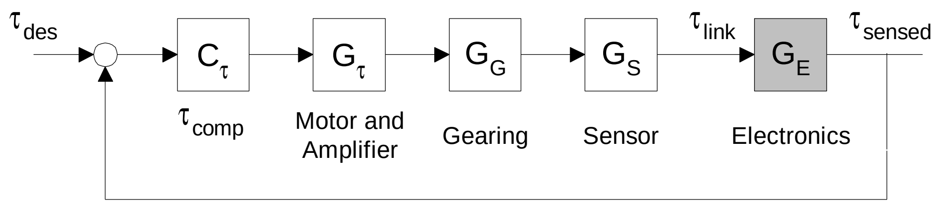

Figure 3-4: Generalized Joint Torque Loop

Once the mechanical sensor converts the applied torque into a mechanical quantity, the next step is to select an appropriate transducer to convert the mechanical quantity into an electronic quantity.

3.3.1 Requirements for Ideal Transducer

A list of the characteristics for an ideal sensor provides an excellent roadmap for understanding the engineering tradeoffs involved in transducer and decoder selection for the mechanical torque sensor. The following characteristics are required for an ideal direct transducer:

- **Infinite throw and infinitesimal resolution**
  The sensor would be able to sense any amount of torque applied
- **Infinite bandwidth**
  The sensor would be instantaneously capable of reporting the amount of torque applied
- **Infinite signal-to-noise ratio**
  The sensor would communicate the value of torque applied without disturbances
- **Simple installation and alignment**
  The transducer would allow for easy installation and alignment in normal operation

In addition, the specifications for the ARTISAN manipulator require the decoder to include a *digital format* to interact with the overall control system. Therefore, additional characteristics include:

- **Infinite length digital word**
  The digital format provides infinite resolution on the digital representation of the information. This requirement is often analogous to the infinite resolution characteristic described earlier.
- **Instantaneous conversion**
  The analog-to-digital conversion takes no time and incurs no latency. This characteristic is analogous to the infinite bandwidth described earlier.

In practice, these ideal characteristics are generally impossible to attain. Hence, tradeoffs occur between the different characteristics, such as higher resolution for lower bandwidth.

3.3.2 Transducer Selection

Table 3-2 lists the characteristics for a variety of displacement and strain transducer options.

Technology	Resolution	Throw	SNR	Format
Strain Gauges	High	Short	Low	Analog
Inductive Transducers	High	Short	Medium	Analog
Piezoceramics	Medium	Short	Low	Analog
Fiber Optics	High	Short	High	Analog
Magnetic Hall Effect Sensors	Medium	Medium	Medium	Binary
Linearly Varying Differential Transformers	High	Long	High	Analog

Table 3-2: Transducer Comparison

From this listing of transducers, the Linearly Variable Displacement Transducer (LVDT) is chosen.

- provide infinite resolution on displacement being limited only by the converter technology and signal-to-noise ratio

- are shielded to electromagnetic noise
- are consistant, tunable and not prone to breakage
- come in variable throw lengths
- and maintain a large linear region of sensing in comparison to the displacement distance.

Further discussion regarding the operation of LVDTs can be found in Appendix A.

Once the transducer is selected, the next step is to choose the proper converter for the transducer.

3.3.3 Converter Selection

LVDTs generate an analog voltage signal to determine the position of the core. This signal can be approximated as a single side-band amplitude modulated (AM) signal. The position information is encoded in the amplitude and phase of the carrier wave with respect to the reference signal. A number of technologies were examined and tested and are listed in Table 3-3.

Technology	Resolution	Bandwidth	3 dB point	SNR	Format
Signetics NE5521	10 bits	Static	125 Hz	Medium	Analog
Analog Devices AD598	10 bits	Static	100 Hz	Medium	Analog
Analog Devices AD2S54	12 bits	Static	125Hz	High	Digital
Analog Devices AD2S93	14 bits	Dynamic	1250Hz	High	Digital

Table 3-3: Comparison of Converter Technologies

In order to decode this signal as well as provide an interface between the sensor and the computer as specified in Section 3.1.4, the Analog Devices 2S93 LVDT-to-Digital Converter was selected.

3.3.4 Converter System Equation

The Analog Devices AD2S93 LVDT-to-Digital Converter incorporates an analog tracking loop with a digital output format and combines it into a single package.

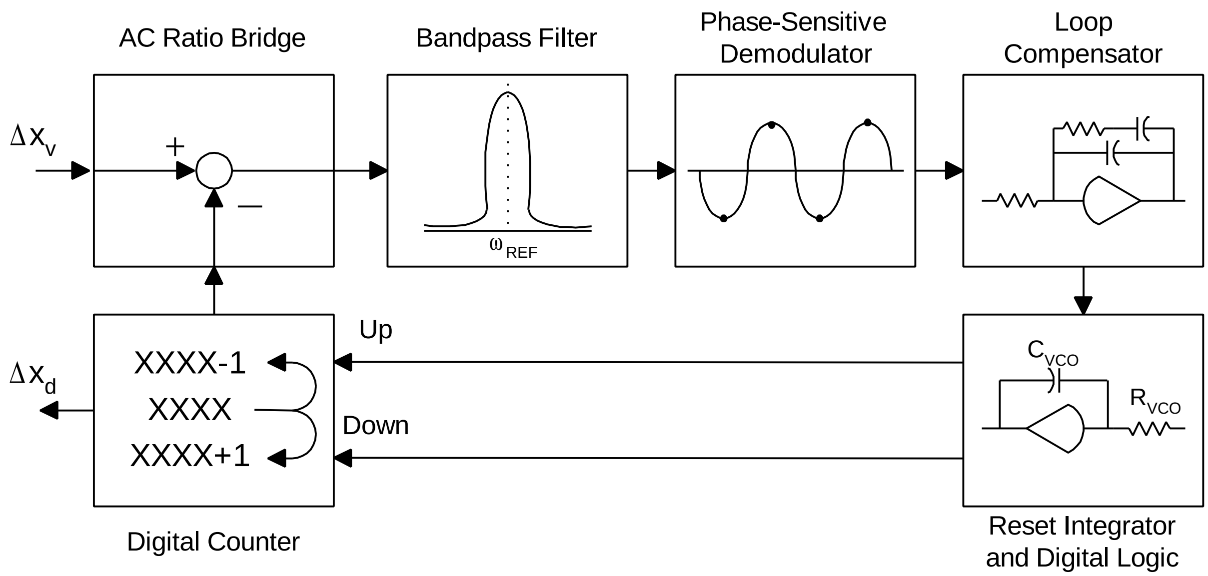

Figure 3-5: Functional Diagram of AD2S93

Figure 3-5 illustrates the sub-components of the AD2S93. The LVDT's secondaries are fed into the inputs of the AD2S93 and passed into the AC Ratio Bridge. There, the ratio of the difference of the two signals over their sum is created and is compared to the estimated digital value of the signal. The resulting error is tracked via the internal loop of the chip which demodulates the position information from the error signal and generates a digital representation through the voltage-controlled oscillator (Reset Integrator) and Digital Counter. The resulting digital word is then available for retrieval from any digital interface through the buffers onboard the AD2S93. A more in-depth discussion of the sub-components of the AD2S93 is found in Appendix C.

3.3.4.1 Simplified System Equation

The system dynamics of the AD2S93 are programmable through selection of the resistors and capacitors that make up the suggested Loop Compensator sub-component. If the subsystems are modeled in the demodulated domain (low frequency range), then the functional block diagram of the AD2S93 can be simplified into the form found in Figure 3-6.

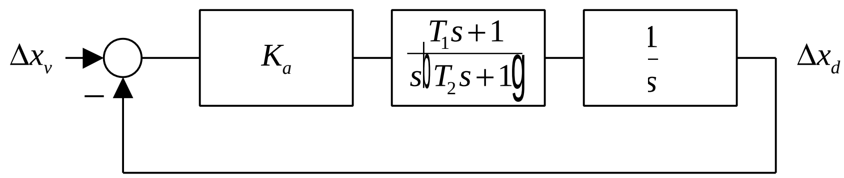

Figure 3-6: Simplified Block Diagram for AD2S93

Using this simplified structure, the transfer function between the input displacement voltage Δx_v and the digital word x_d of the AD2S93 can be approximated by Equation (3.22)

$$\frac{\Delta x_d}{\Delta x_v} = \frac{T_1 s + 1}{\frac{T_2}{K_a} s^3 + \frac{1}{K_a} s^2 + T_1 s + 1} \tag{3.22}$$

where the values of T_1, T_2 and K_a are determined by the discrete components of the Loop Compensator specified by Analog Devices.

Further discussion of the simplified system model and expressions for the discrete components in terms of performance can be found in Appendix B.

3.3.4.2 Enhanced System Equation

Closer examination of the system's components and a modification of the Loop Compensator provides a simplified expression for the AD2S93's system equation. As discussed in Appendix C, the enhanced system equation for the AD2S93 is expressed in Equation (3.23).

$$\frac{\Delta x_d}{\Delta x_v} = \frac{K_{AC} K_{BP} K_{PSD} K_{INT}}{(s + w_{BP})(s^2 + ps) + K_{AC} K_{BP} K_{PSD} K_{INT}} \tag{3.23}$$

where the variables are defined in Appendix C.

Using Equation (3.23), the components of the AD2S93 are chosen to provide performance characteristics that benefit the overall torque loop design. Equation (3.23)is used extensively in determining the performance of the sensor electronics.

3.4 Summary of System Equations

All of the system components have been analyzed and Table 3-4 summarizes the system equations for each of the components.

Subsystem	System Equation	Bandwidth
Current Amplifier	$\frac{i_m}{v_c} = \frac{R_{43}C_{17}s+1}{R_1C_{17}L_m s^2 + \left(R_1R_{43}R_m + \frac{10}{14}R_{43}C_{17}\right)s + \frac{10}{14}}$	1Khz
Motor	$\frac{t_m}{i_m} = K_T$	Infinite
Sensor and Gearing	$\frac{t_s}{t_m} = \frac{Nw_s^2}{s^2 + 2z_s w_s s + w_s^2}$	w_s (approx 130 Hz)
Transducer	$\frac{\Delta x_v}{t_s} = \frac{k_s}{116.9 r_s}$	Infinite
Converter	$\frac{\Delta x_d}{\Delta x_v} = \frac{K_{AC}K_{BP}K_{PSD}K_{INT}}{\left(s+w_{BP}\right)\left(s^2+ps\right) + K_{AC}K_{BP}K_{PSD}K_{INT}}$	Up to 1.25 kHz

Table 3-4: Summary of System Equations

By comparing the bandwidths of the components listed in Table 3-4, the mechanical sensor is the limiting component of the open-loop system, as long as the AD2S93 system is programmed to track at a very high bandwidth (on the order of 1 kHz). If the converter is not capable of this performance, the dynamics of the converter will have to be included in any control law developed.

Chapter 4 develops the constraints for the joint torque loop and the control law necessary for both the AD2S93 and the overall joint torque loop.